Kiril Datchev
MA 495/595
Spring 2024

Homework 9

Due March 21st on paper at the beginning of class. Please let me know if you have a question or find a mistake. There are some hints on the second page.

1. (a) Let u and φ be C^2 near $[a, b]$. Use integration by parts to derive a formula for $\int_a^b \varphi u'' - u\varphi''$ analogous to Green's second identity (Theorem 2.11).
   (b) Use this and a good choice of φ to prove a version of the mean value formula (Theorem 9.3) with $B(x_0; R)$ replaced by $[-1, 1]$.
   (c) You don't have to hand anything in for this but you may enjoy thinking about what is the equivalent of Corollary 9.4.
2. Borthwick Exercise 9.2.

Hints:

1. For part (b) use $\varphi(x) = |x| - 1$. For part (c) note that a one-dimensional ball is an interval, and its boundary consists of two points. The 'integral' of a function f over a pair of points p and q is $f(p) + f(q)$.
2. This is an expanded version of a classic proof with no equations: `https://www.ams.org/journals/proc/1961-012-06/S0002-9939-1961-0259149-4/S0002-9939-1961-0259149-4.pdf`.

   Here is a picture of the balls $B(0, R)$ and $B(x_0; R)$. The point is that they get closer and closer to coinciding as $R \to \infty$.